NEWTON-RAPHSON MPC CONTROLLED ACTIVE VIBRATION ATTENUATION

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ABSTRACT
This article describes the computationally efficient predictive control of an active structure with bonded piezoelectric transducers. A highly under-damped physical structure, such as a helicopter rotor blade is represented by a simplified laboratory model. After introducing the experimental hardware the identification procedure is described. Theory behind the Newton - Raphson MPC algorithm is summed up, and its laboratory implementation and experimental verification illustrated. Results of the experiments show, that the proposed predictive control approach is capable of achieving high sampling rates in practical use while still respecting process constraints and maintaining guaranteed stability.

KEY WORDS
MPC, efficient, invariant sets, active vibration damping, smart material, NRMPC

1 Introduction
Undesired vibration is present in countless real life engineering applications, and raises economic, quality and safety issues. Although some level of inherent physical damping is always present, this most of the time is not satisfactory enough [1]. To decrease the mechanical response of structures driven at or near the resonant frequencies, it is possible to employ vibration canceling techniques. Passive means of vibration attenuation may not always be viable due to the limited effectiveness in lower bandwidths, and the fact that involves manipulation with the mass and stiffness properties of objects. There are several means of semi-active damping procedures currently under research or already in use. Properties of semi-active treatments mostly depend on the actual application: for example may involve shunted or electrically tuned piezoelectric wafers, magneto - rheological dampers and similar devices.

Active vibration attenuation methods on the other hand utilize actuators to exert force effects on the controlled object. These actuators are driven via algorithms which gain their feedback signal from sensors. Individual elements of the active vibration attenuation systems are highly integrated with the end product and may be regarded as one complex mechatronic unit. Frequently the term ”smart material” is used to refer such and similar products.

Often, there is very little attention devoted to the control algorithm governing the behavior of the actuators in active vibration damping applications, or quite the contrary: the algorithm design does not take into account the properties of the physical system. The obvious basic control methods are well investigated, although may not ensure the necessary performance, or not even guarantee stability. Generally simple PID or linear quadratic control is considered, the deployment of more intricate algorithms is prohibited due to the high sampling rate nature of the application.

Every actuator has its natural limits, presenting performance or safety boundaries. The only current control technique capable of actively handling such constraints is model predictive control (MPC), which utilizes a state-space mathematical model to obtain future optimal control signals. The use of MPC in a vibration attenuation application has been demonstrated in [2], where the algorithm has been transcribed directly into machine language code and implemented on a dedicated microcontroller. This allows the use of high sampling rates in the real time application, although fails to ensure guaranteed MPC stability.

This article demonstrates the possibility to use a computationally efficient algorithm described in [3], [4] and [5] which is capable of handling constraints and ensuring stability as well. We shall refer to this method as Newton - Raphson MPC (NRMPC) as the on-line solution is based on the iterations of the well known numerical algorithm. Although the underlying theory behind NRMP is somewhat intricate, describes a rather elegant and computationally simple on-line solution, which allows previously unmatched sampling speeds.

The compound process of creating a suitable experimental laboratory model, its modeling, identification and the implementation details of NRMPC will be described in this article. Results of the experiments will show the viability of this particular control method in active vibration attenuation.
2 Laboratory setup and Identification

The classic clamped cantilever beam with a free end illustrates numerous real life applications. Helicopter rotor blades are excellent examples, as their vibration tends to influence flight dynamics and fuel economy. Other similar large, underdamped structures can be found in the space industry: antenna masts, solar panels or manipulator arms. Both the active damping of helicopter rotor blades and space structures using piezoelectric wafers are an attractive field of research. (see e.g., [6]) The clamped cantilever example was chosen to be as the proving ground for the NRMPC and other efficient algorithms. The model shall demonstrate active damping by minimizing the beam tip vibration, which is subject to external excitation.

The experimental device was designed by taking into account future applications: these defined the inclusion of five transversal bending modes under 500 Hz. To cover the bending modes within the mathematical model, this would indicate the need to use a high order state space model in the MPC algorithm, requiring at least a sampling rate of 5000Hz or more. If one considers only predictive control, the high sampling rate requirement demands an efficient algorithm. The high order model with fast sampling on the other hand may prohibit the use of multi-parametric MPC with guaranteed stability and feasibility of constraints, since it would involve intractable off-line computation times and possibly enormous memory requirements. This leaves NRMPC as a very good candidate control system.

2.1 Experimental Hardware

The material, shape and the placement of the piezoelectric actuators have been determined by successive finite element modeling (FEM) simulations, taking into account the original design considerations and the bandwidth of interest [7]. These simulations captured the complex electromechanical interaction between the transducers and the beam material. Both static and dynamic FEM tests were carried out, including modal, harmonic and transient simulations. Results of these not only have contributed to the physical design, but the harmonic excitation simulation provided an initial model for the control system development.

Beam material was chosen to be commercially available 99.5% pure aluminum, with one end clamped to a support structure, the other left to vibrate freely. The beam has the dimensions of 550 × 40 × 3mm and its material designation is EN AW 1050A.

Figure 1. shows four MIDÉ QP16 piezoelectric transducers bonded on the beam surface with two component, high strength epoxy glue. The current application utilizes only the two closest to the clamped end as actuators, to maximize beam tip displacement. These two actuators are marked as “PZT1.2” on the figure and are electrically connected as counter - phase. (i.e., with opposing polarity)

The rest of the transducers are short circuited to prevent any electro-mechanical interaction with the structure.

The actuator mode transducers are connected to a MIDÉ EL-1225 power amplifier with 20 V/V gain, which in turn driven by a control signal coming from a measuring card. In addition to the D/A conversion of the control signal, the high speed 16 bit National Instruments PCI-6030E measuring card is also responsible for the A/D conversion of the feedback signal.

Current laboratory device assumes a feedback signal on the beam tip displacement to be measured directly by an industrial, precision LASER triangulation device, of the make KEYENCE LK - G 82 marked as “LASER 1” on the figure. This is complemented by a proprietary filtering and processing unit and a settings software.

The control algorithm is running real-time on an xPC target computer. The measuring card is located in this computer, and its sole responsibility is to use its resources for the on-line part of the NRMPC algorithm. Control code is loaded onto the target computer via Ethernet from a host computer. This host computer is running the Matlab / Simulink suite and control software for the triangulation device.

2.2 Measurement and System Identification

To obtain the state-space mathematical model for the beam tip deflection, three methods were considered. Developing an explicit solution would be a favorable solution, although its complexity is beyond the scope of this project. Despite the formidable of the development of a reasonable model, showing the interconnected electro-mechanical behavior, several works attempt to utilize it [8]. A further way to obtain the model is to match real life measurement data to the FEM model, and use it to acquire the state-space matrices from this. Harmonic simulation results were used to create state space models in [9].

The most direct approach is to use real life experimental measurements, utilizing Matlab / Simulink Real Time Workshop and xPC target, to establish a mathematical model. Although identification of a correct state-space model is essential to the proper functionality of any MPC,
A Matlab / Simulink generated chirp signal was chosen as to generate the driving force of the piezoelectric transducers. The measuring card output provided a signal with peak amplitude of $\pm 5 \, \text{V}$, which was further amplified to the maximal possible voltage level on the actuators, which is $\pm 100 \, \text{V RMS}$. The maximal possible voltage level is required to minimize the signal to noise ratio. This is especially important in between the resonant frequencies.

Two distinct experiments were carried out: one to get a second order and one a higher order state space model. The second order model covers only one vibration mode, therefore the measurement is straightforward. On the other hand the higher order model must cover the whole bandwidth of interest. Since the LASER triangulation sensors have limited precision for a given deflection span, the amplitude / voltage gain was re-set at each measurement pass. To improve precision, digital low-pass filtering was enabled according to the actual frequency span. Measurement sampling rates were 0.001 seconds and 0.0002 seconds for the second order and 0.0002 (5kHz) for the high order model.

One has to consider an additional problem with this approach. The chirp signal must pass through resonances at a slow enough rate, in order to obtain realistic deflection levels. This naturally brings the need to use a high number of measurement points, which all have to be stored real time. The xPC target system provides an on-line measurement sample storage capacity, which is limited by the computer RAM. The given system allowed the storage of approximately 14 million samples, along with the time series data. Each partial measurement has been retrieved and converted into Matlab using xPC specific commands [10]. The resulting raw time series data contained 13.5 million samples. Due to the data set size, preparation, identification and post processing proved to be a rather difficult task on a personal computer conforming to up to date standards.

System identification from the raw measured data has been carried out using the Matlab System Identification Toolbox. After signal detrending and removing means, fast Fourier transformation was used to convert time series data to the frequency domain. To cut off unnecessarily high frequencies and reduce the amount of working data, the data set was subjected to low-pass filtering. The size of the frequency domain data would still prohibit the direct application of identification routines, therefore frequency response and spectrum was estimated using spectral analysis with frequency-dependent resolution returning. Spectral analysis included 2000 discrete points up to 500Hz. The data set prepared with this pre-processing technique is suitable for direct identification.

After pre-processing, both the second order and the higher order state-space model have been obtained using the subspace iteration method implemented as default in the toolbox [11]. Taking into account the bandwidth of interest and the singular values of the possible state-space models, a model order of $n_x = 12$ has been chosen in addition to the simpler second order which is described by:

$$A = \begin{bmatrix} 0.9981 & -1.2233 \\ 0.0021 & 0.9985 \end{bmatrix}, \quad B = \begin{bmatrix} 6.697 \times 10^{-6} \\ -8.001 \times 10^{-6} \end{bmatrix}, \quad C = \begin{bmatrix} -0.5774 & -0.7069 \end{bmatrix} \quad (1)$$

where the output of the model is beam tip deflection in millimeters, and input is actuator voltage.

Figure 2. compares model response in the frequency domain for the second and the twelfth order state space systems with actual measurements. Due to large number of samples, the high order model is compared to the frequency function generated from the actual measurement data. Both models show good agreement with the physical system, capturing the resonant behavior of the beam tip. Although the more complex model contains higher order dynamics and represents the beam tip behavior more precisely, for the purposes of control algorithm implementation the second order state-space system is satisfactory enough and has been considered in experiments.

3 NRMPC

The pre-stabilized closed loop formulation of the original MPC problem and the subsequent elegant on-line algorithm
originally introduced in [3] promises computational efficiency, albeit with the sacrifice of a small degree of optimality. While this section may give only a superficial insight to the problem, it is nonetheless important to introduce the NRMPC algorithm. Let us consider a linear time-invariant state-space system of order \( n_x \), controlled by a fixed feedback law \( u_k = K x_k \):

\[
x_{k+1} = A x_k + B u_k \quad y_k = C x_k
\]  

(2)

Instead of directly optimizing the system input like in the case of quadratic programming based MPC (QPMPC), the so called perturbation vector \( f = [c_k c_{k+1} \ldots c_{n_c}]^T \) is introduced, containing a number of free variables equal to the prediction horizon \( n_c \). In the absence of constraints the pre-stabilized loop can be optimal in some sense, this implementation for example considers a linear quadratic (LQ) feedback with state penalization \( Q \geq 0 \) and input penalization \( R \geq 0 \). When constraints are inactive, perturbations are zero, however during transients this loop is no longer optimal - perturbations will change to a nonzero value. The formulation describing control moves, and the closed loop system are:

\[
u_k = K x_k + E f_k \quad x_{k+1} = \Phi x_k + B c_k
\]  

(3)

where vector \( E \) is responsible for selecting only the first element from the perturbation vector and \( \Phi \) denotes the dynamics of the closed loop fixed state feedback system: \( \Phi = (A + BK) \). The dynamics of the augmented system is then described by the following autonomous state space system:

\[
z_{k+1} = \Psi z(k) \quad \Psi = \begin{pmatrix} \Phi & B E \\ 0 & T \end{pmatrix}
\]  

(4)

where \( z = [x \ f]^T \) is the augmented state vector, 0 is a matrix of zeros of conforming size. In the simplest case \( T \) is a shift matrix with ones on the sub diagonal, and zeros elsewhere. If we consider symmetric constraints on the control signal, the conditions for feasibility as applied to the augmented ellipsoidal set defined by \( E_z = \{ z | z^T Q_z z \leq 1 \} \) are:

\[
\begin{bmatrix} d^2 & [ K \ E ] \\ * & Q_z \end{bmatrix} \geq 0
\]  

(5)

where \( * \) denotes the symmetric part of the matrix. Vector \( d \) contains the symmetric input constraints in the form \( |u| \leq d \). For practical reasons, it is good to impose bounds on the predicted cost along the trajectories of the autonomous system. For a given bound \( \gamma \) the cost value (see (11)) \( J \leq \gamma \) is ensured for all initial conditions of the autonomous system in \( E_z \), if the invariance condition is:

\[
Q_z - \Psi^T Q_z \Psi > \frac{1}{\gamma} \begin{bmatrix} C^T & K^T \\ 0 & E^T \end{bmatrix} D \begin{bmatrix} C & 0 \\ K & E \end{bmatrix}
\]  

(6)

where \( D = \text{diag}(I, R) \) and \( I \) is the identity matrix. As it is evident from the development of the NRMPC algorithm in [5], the size of the stabilizable set can be maximized, if \( T \) and \( E \) also become optimization variables. To preserve optimization convexity, one has to perform a nonlinear transformation of variables on the invariance and feasibility conditions. Consider the following identities:

\[
Q_z = \begin{bmatrix} X^{-1} & X^{-1} U \\ X^{-1} U^T & \bullet \end{bmatrix} \quad Q_z^{-1} = \begin{bmatrix} Y & V \\ V^T & \bullet \end{bmatrix}
\]

\[
N = UTV^T \quad M = EV^T
\]

where \( \bullet \) denotes the blocks of \( Q_z^{-1} \) and \( Q_z \) which are determined uniquely by \( X, Y, U \) and \( V \). After applying a nonlinear congruence transformation, conditions for invariance (6) and feasibility (5) will be transformed to:

\[
\begin{bmatrix} \gamma I & 0 & D^{1/2} \begin{bmatrix} C Y & C X \\ K Y + M & K X \end{bmatrix} \\ * & \begin{bmatrix} Y & X \\ X & Y \end{bmatrix} \begin{bmatrix} \Phi Y + BM & \Phi X \\ N + \Phi Y + BM & \Phi X \end{bmatrix} \\ * & * \end{bmatrix} \geq 0
\]  

(7)

\[
\begin{bmatrix} d^2 & [ K Y + M \ K X ] \\ * & \begin{bmatrix} Y & X \\ X & Y \end{bmatrix} \end{bmatrix} \geq 0
\]  

(8)

On the contrast to the formulation with the unoptimized matrices \( T \) and \( E \), this case the initial stabilizable set is not dependent on the control horizon. Since \( Y \), representing the projection of the augmented hyper ellipsoid into the state-space is independent of horizon length, it is sufficient to choose \( n_c = n_x \). The matrices \( T \) and \( E \) may be computed finally from:

\[
T = U^{-1} N V^{-T} \quad E = M V^{-1}
\]  

(9)

It is necessary to maximize the volume of the projection of the augmented invariant ellipsoid \( E_z \) into the \( x \) subspace, which defines the initial stabilizable set. On the other hand the intersection of \( E_z \) with the \( x \) subspace defines the space, where the optimal control is feasible, without the need to calculate perturbations. The projection is defined by the matrix \( Y \) and the intersection by \( X \).

The off-line algorithm in a prediction dynamics optimized NRMPC will contain the following general steps [5]:

**Algorithm 3.1**

**Off-line procedure:**

- Calculate a feedback matrix \( K \) which is optimal in some sense, while ignoring constraints.

- The maximization of the volume of the augmented invariant ellipsoid set projection and intersection with the state-space is performed, by solving the SDP:

\[
\max \left( -\log \det \begin{bmatrix} Y & 0 \\ 0 & X \end{bmatrix} \right)
\]  

(10)
subject to conditions (7) and (8), in the optimization variables \( X, Y, N, M \).

- Factorize \( X \) and \( Y \) to determine \( U \) and \( V \).
- Using relation (9) for \( N \) and \( M \) solve for \( T \) and \( E \)...
- Utilize relevant results as parameters in the on-line NRMPC algorithm.

Now after creating a feasible and invariant augmented hyper ellipsoid in the augmented state-space, let us describe the on-line optimization routine [4]:

**Algorithm 3.2**

*Online procedure:* At each sampling instant \( k \) perform the following minimization procedure:

\[
\min_{f} f_k^T f_k \quad \text{s.t.} \quad z_k^T Q_z z_k \leq 1 \tag{11}
\]

The actual control signal is calculated and the procedure is repeated at the next sampling instant.

The optimization constraint in (11) may be rewritten, by utilizing the partitioning of \( Q_z \) as follows:

\[
z_k^T Q_z z_k = x_k^T \hat{Q}_{xx} x_k + 2 f_k^T \hat{Q}_{fx} x_k + f_k^T \hat{Q}_f f_k \leq 1 \tag{12}
\]

The optimization problem with the transformed constraint (12) can be understood as a simple geometrical problem: it is actually the search for the shortest distance of an ellipsoid surface from the origin. In case the system origin is included in the ellipsoid defined by the constraints, there is no need to perform the optimization the problem is already optimal. Otherwise to find the optimum of this problem, one may use Lagrange’s constrained optimization method. There is only one viable solution at a given time sample for \( \lambda \), since the gradient must be negative at one tangent point:

\[
f = \lambda M \hat{Q}_{fx} x_k \tag{13}
\]

\[
\Phi(\lambda) = \hat{Q}_{xf} [M \hat{Q}_{f}] M - \hat{Q}_{ff} \hat{Q}_{fx} x_k + x_k^T \hat{Q}_{xx} x_k - 1 = 0 \tag{14}
\]

where \( M \) is defined as \( M = (I - \lambda \hat{Q}_{ff})^{-1} \) and \( \lambda \) is the unique real root of \( \Phi(\lambda) \). One may utilize the Newton-Raphson (NR) root searching algorithm to calculate the unique real root of (14).

It is easy to see the efficiency behind this approach, since there are no complex matrix operations in the on-line run. All it is needed to find the perturbation vector is the root of \( \Phi(\lambda) \) by the Newton-Raphson algorithm. Since all the derivatives of this expression are positive for all \( \lambda \leq 0 \), the Newton-Raphson method will converge quadratically when initialized with \( \lambda = 0 \). In practice no more than 20 iterations are needed for this. Other tactics, such as making use of the eigenvector - eigenvalue properties of \( \hat{Q}_{ff} \) or concatenating matrix operations shall further increase the operational speed. It is possible to show that computational time grows linearly with the system order considered.

### 3.1 NRMPC Code Implementation

Initially the whole NRMPC algorithm has been implemented in the high level scripting language of Matlab for simulation and testing purposes. Later stages of the development required a C language S-Function of the on-line part, which is capable of running under Simulink and xPC target as well. In addition to the block containing the on-line part of the NRMPC code, the Simulink scheme also includes a block for a Kalman state observer. To speed up development, this observer has been chosen from the Signal Processing blockset. To increase computational speed, further implementations shall integrate this into one compact on-line unit with the NRMPC code.

All the matrix multiplications have been thoroughly reviewed and concatenated in order to minimize computational time. Unnecessary operations were removed from the loops, optimizing C code efficiency in order to achieve high computational speeds. Matrix operations are performed by BLAS, see 3.1.1.

Simulations shed light to the fact, that the extension to the NRMPC algorithm described in [4] is not necessary for the cantilever-beam example. This extension would theoretically improve on the optimality of the solutions, matching closely that of the QP solution. But as several tests with the second order system demonstrated, unlike in the case of over-damped systems the extension did not scale the perturbations to significantly lower values. Therefore this extension has not been implemented in the C version of the on-line controller, to avoid unnecessary computations.

Although it is theoretically possible to use the NRMPC controller based solely on [3], in the case of the cantilever beam example (and plausibly other under-damped systems) this would be very prohibitive. The volume of the initial stabilizable set using this approach is too small, when one considers the possible range of deflections at the beam tip. This is even true, when one increases the prediction horizon to very extreme lengths. To render the algorithm practically useful, one has to enlarge the set - at which the controller may operate. This is precisely what the optimization of prediction dynamics introduced in [5] does, without changing the efficiency of the on-line algorithm. We can conclude, that for this and similar systems it is necessary to use optimized prediction dynamics to preserve practical usability.

### 3.1.1 Matrix operations in C

The on-line code of the NRMPC algorithm is essentially simple, it does not include complex matrix operations. To increase speed and code re-usability, the matrix - matrix and matrix - vector multiplications and other operations are performed through BLAS. BLAS stands for Basic Linear Algebra Subroutines and it is an algorithm package responsible for the mathematical operations in numerous scientific and engineering applications. BLAS functions can be directly called as C subroutines, provided that the library is
present at compiling.

BLAS is platform and processor architecture dependent. Matlab itself uses it for all the fundamental operations, and it is possible to compile and use BLAS subroutines under the Windows platform. (This is also true for LAPACK - a linear algebra package.) However the problem arises when the BLAS library compiled for Windows needs to be transferred to the xPC target computer, which uses an essentially DOS - like environment. On the contrary, if the library would be compiled under and for the use of a UNIX or DOS system, it would not be possible to compile the S-Function in Matlab.

There is a convenient trick at hand to solve this dilemma. The key to the solution is Cygwin, a collection of tools, command line and programming interface resembling Unix and running under Windows. After installing Cygwin, the BLAS and LAPACK packages provided and the GCC compiler, one has to recompile the libraries from source. Functionality of one error reporting routine has to be disabled, to avoid compatibility issues. In addition to this one has to specify that the libraries must be able to run outside Cygwin in the appropriate configuration file. Libraries created this way will be both compilable as S-Functions and will run under xPC.

3.1.2 Numerical precision issues

Prior to its implementation onto the physical system, the NRMPC code has been extensively tested in simulation. State-space models of over-damped systems behaved as expected: invariance and feasibility of the constraints has been preserved. However when under-damped systems were considered, such as the model of the cantilever beam, a violation of the invariance condition occurred.

In case the size of the maximal stabilizable set is not limited in the off-line procedure, some elements of the definition matrices grow to extreme proportions. In fact, the on-line run produces perturbation vectors with unrealistically large elements, which are then multiplied by the appropriate matrix parameter. This parameter contains extreme small elements, by which we get the final perturbation value at the given time step. This process is in fact prone to numerical problems.

This violation of invariance arises due to numerical issues. There is a maximal possible volume and a corresponding cost limit $\gamma$ for the initial stabilizable set, under which the algorithm is reliable. The maximal set size can be increased through overriding the off-line SDP solver defaults. In this particular implementation SeDuMi has been utilized to calculate the the controller parameters. Superceding its default precision settings may enlarge the volume of the stabilizable set by orders of magnitude. Increasing SDP precision may lengthen the computational time when using higher order systems, but has no effect on the efficiency of on-line computations whatsoever.

4 Experimental Results

In order to test the effects of the NRMPC control on the vibration properties of the system, a 5 mm initial deflection of the beam tip was considered. The control algorithm utilized a second order model to generate its predictions. Since the augmented system dynamics has been also optimized according to [5], the prediction horizon was chosen to be the sufficient minimum recovering the maximal admissible set under the given feedback law: equaling the model order $n_c = n_x = 2$. The fixed LQ feedback considered a state penalization $Q = C^T C$, while the input penalization $R = 1E^{-4}$ has been chosen with sufficient performance and the constraints in mind.

The beam was deflected to its initial position, then left to vibrate freely - without any further outside force effect. The time domain response without control (i.e., free vibration) and with NRMPC control is demonstrated on Figure 3. The settling time of the uncontrolled beam tip vibration is in the order of minutes, marked with a dashed line on the figure. The experiments clearly show, that the greater part of the vibration is practically damped after about 2 - 3 seconds, shown as solid line. Naturally the control settling time depends on the initial deflection.

Figure 4 shows the controller voltage applied to the piezoelectric actuator. Controller voltage has been limited to $\pm 120V$. It is clear from the figure, that the constraints have been well respected, the peak value approaching but not exceeding the prescribed limits.

There is an inherent drift visible in the control signal, a deviation from its zero equilibrium value. This is due to
the nature of the measuring process: the reference zero of the triangulation device is arbitrary. Even if the position of the beam tip would be identical to this point, the vibration and the initial deflection tends to change it - thus the desired equilibrium is shifted, and a constant non-zero control output is generated.

Since the second order model has been assumed throughout the experiment, neither the state-space model nor predictions can be expected to include high frequency effects in the control process.

Sampling frequency was set to 5 kHz with the second order model, the measurements were performed with this setting. A trial with 10 kHz has been successfully performed, demonstrating the excellent efficiency of the method. Naturally a higher order model will require longer computation times, but this may be compensated by better code implementation or different hardware.

Even though NRMPC is a sub-optimal control method, the comparison with different MPC algorithms with stability and constraint feasibility guarantees shows no significant differences in the beam tip response in simulation. Quadratic programming based stable MPC (QPMPC) and multi-parametric MPC (MPMPC) drive the actuators hard into saturation, while the NRMPC control input slightly differs from the former two. These findings have been demonstrated in experiments, while showing other significant advantages of NRMPC over QPMPC or MPMPC with guaranteed stability and constraint feasibility [12].

5 Conclusion

An implementation of the efficient Newton-Raphson Model Predictive Control algorithm on an actively controlled structure has been introduced. In addition to presenting the laboratory device, most focus was devoted to practical realization of the NRMPC controller. The results of the laboratory experiments indicate, that the NRMPC control method is a good candidate for high sampling rate applications not only in theory, but also in practice. A reliable 5 kHz sampling speed has been achieved, suggesting room for further improvement. By sacrificing a negligible degree of optimality, the NRMPC algorithm is able to cope with the high computational speeds, while maintaining feasibility of the constraints and guaranteed stability.

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