Verification of the finite element model of resonant mode shapes of a controlled beam using high-speed video

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Abstract: The paper presents a model of the resonant resonant behavior of a cantilever beam equipped with piezoelectric transducers and contrasts the modal shapes computed using the finite element method (FEM) to high-speed video footage. Non-invasive, non-contact optical methods such as laser Doppler vibrometry (LDV) and electronic speckle pattern interferometry (ESPI) can be used to visualize the mode shapes of vibrating objects. Instead of the abstracted representation of the modal shapes using these methods, this paper proposes the use of high-speed video for visualization. The mode shapes of a clamped cantilever equipped with piezoelectric actuators are contrasted to high-speed video microscopy images. The acquired footage illustrates the mode shapes only in the lowest resonance frequencies, small amplitudes are not clearly visible. According to the results presented in this paper, high-speed video may be used to illustrate modal shapes in select cases, mainly if a visual confirmation of the shapes is required, the expected amplitudes are large and only the lowest mode shapes are in the center of interest.

Keywords: vibration control, finite element modeling, high-speed video, resonance, mode shape

1. INTRODUCTION

Mode shapes due to the resonance phenomenon in vibrating objects have been studied in great detail in history (Youmans, 1873). This intense interest has been gradually transferred to the technological advances, nowadays utilized in high-tech products and everyday objects as well. The means and tools for assessing vibration properties have evolved as well. Currently, the most common means to analyze the vibration properties—in particular the resonant mode shapes—of an object are analytical methods, numerical methods and experimental methods.

The analytical approach may provide an exact solution to a vibration problem; however, it is limited to simple shapes with simple boundary conditions. It is difficult to arrive at the exact solution for objects with intricate geometry and boundary conditions (Inman, 2007). On the other hand, numerical analysis is able to cope with elaborate shapes and result in a good approximation of the mode shape. The disadvantage of the numerical method is that it always contains a degree of uncertainty and may require considerable computing time for complex structures (Hatch, 2000). Experimental devices and procedures aimed at assessing and visualizing modal shapes come in many variations. One of the most common experimental techniques is exciting the structure artificially (e.g. modal shaker) then making acceleration measurements at discrete points (e.g. accelerometers). This procedure allows the reconstruction of modal shapes only with a limited certainty.

Recent technological advances transferred new tools to the realm of vibration engineering. In this article, we will focus on non-invasive, non-contact optical methods aimed at vibration analysis. One of the best known optical methods is Electronic Speckle Pattern Interferometry (ESPI) (Butters and Leendertz, 1971), in which a laser beam is used in conjunction with electronic imaging equipment and subsequent data processing to visualize vibration patterns. In out-of-plane vibration measurements, a laser beam is divided by a beam splitter with one beam directed at the specimen and the other at a reference. Both these beams are reflected and combined into a sensor of a CCD camera. The resulting interferogram is stored as a reference and then the next image frame is subtracted from it. The process creates a fringe pattern, showing the mode shape of the vibrating object. One of the other advanced optical methods used in vibration engineering is laser Doppler vibrometry (LDV) (Halliwell, 1979). In a LDV device, a laser beam is splitted to create an object and a reference beam. In the object beam, a laser beam is splitted to create an object and a reference beam. The object beam is directed at a measured surface, then an interferometer measures the difference between the reference and the reflection. Vibration velocity is estimated based on the Doppler shift of the beam frequency. Although the nature and working principle of LDV devices implies point-like measurements, a conventional motor-driven mirror and software processing allows the reconstruction of displacements measured discretely along a dense mesh (scanning-LDV or SLDV) (Maddux, 1993). The comparison of the ESPI and LDV methods is given in the work of Ma and Li (Ma and Lin, 2005),
where the authors analyze the vibration characteristics of a piezoceramic transducer.

We will shift our focus from the above-mentioned indirect optical methods to something else: video cameras. Filming fast events is a natural way to visualize movement that is normally hidden to the human eye. Using high-speed footage has the advantage of showing motion as it; the real event without the abstract representation of ESPI and LDV devices. High-speed video equipment is used extensively in science; applications include chemical engineering (Nakamura et al., 2007) meteorology (Montanyà et al., 2011), medicine (Lohscheller et al., 2007) and others. High-speed video cameras are employed in mechanical and vibration engineering as well. Yogeve et al. studied the buckling effect on a beam-mass system (Yogeve et al., 2007), Helfert et al. analyzed magnetic bearings in the case of a failure (Helfert et al., 2006), while others measured impact rebound height (Li and Darby, 2006) or the velocity of a rotating container (Dragomir et al., 2012) using this method. However, examples of visualizing mode shapes by using a high-speed camera are rare to find in academic literature.

In this article, a high-speed video microscope is used to film the modal shapes of the clamped beam with piezoelectric actuation, which featured in Figure 1. The video footage illustrates the mode shapes and the resulting deformation of this smart beam. We will describe the instrumentation, the experiment and our results in the upcoming discussion. This paper will also provide a brief description of the presented challenges.

2. INSTRUMENTATION

2.1 Smart cantilever beam

The smart structure used in the experiments featured in this paper is an aluminum cantilever beam actuated by piezoceramics (Takács and Rohaf-Ilkiv, 2012). This simple setup is often used to demonstrate active vibration control concepts in numerous academic works. A clamped cantilever beam may conceptually represent the typical dynamic properties of a wide range of real-life structures (Chiang and Safonov, 1991) like robotic manipulators (Kang and Mills, 2005), helicopter rotors (Lu and Meng, 2006), aircraft wings (Amer and Bauomy, 2009), etc. The structure considered here is made of EN AW 1050A type aluminum, measuring 550 × 40 × 3 mm for its length, width and thickness respectively. The beam contains four bonded piezoceramic actuators, of which only the two mounted 12 mm from the clamped edge are enabled during the experiments. The other two actuators are short-circuited, so that the reciprocal electromechanical action with the structure is contained. The type of the actuators is Midé QP-16n and are made from PZT5A material.

2.2 High-speed video microscope

The resonant mode shapes of the beam were filmed using a Keyence VW-9000 series high-speed video microscope system, featured in Figure 2. This system consists of a main unit and a camera head. The type of the main unit used in the experiments was a Keyence VW-9000E, containing a built-in display, a computer, camera controls and an integrated light source. Frames captured by the camera head are saved onto a 8 GB semiconductor memory, which is then transferred to the 500 GB hard drive of the main unit (Keyence Inc., 2012). The integrated light source uses a 80W high-brightness metal-halide lamp for monochrome video and is connected to the optics mounted on top of the camera head using an optical fiber cable. The main unit also contains a 14 bit analog input terminal with selectable ±10, ±5 and microphone signal, which can be used to synchronize sensor data with the video file.

The Keyence V-600M camera head used for this work was equipped with a 1/2" monochrome CMOS sensor with a 640 × 480 px (VGA) native resolution (Keyence Inc., 2012). The beam is relatively large compared to the usual targets of this system and in order to gain more light for a given pixel, a monochrome camera head was favored in comparison to a color camera one. The maximal native framerate of the device is 4000 frames per second (FPS), however, this may be increased up to 230 000 FPS with more powerful lighting and sacrificing pixel resolution.\(^1\) Electronic shutter is user selectable, ranging from 1/30 to 1/900 000 s.

The camera was connected to a Keyence VW-Z5 long-range macro zoom unit with projection optics for the light source. The zoom unit is capable of focusing in the observation range from 200 mm with a horizontal view size of 19.9 mm (wide) to 5 mm (tele), up to a range of 2000 mm with a horizontal view size of 284.4 mm (wide) to 71.0 mm (tele) (Keyence Inc., 2012). The light coverage

\(^1\) Having 160 × 32 px resolution at 230 000 FPS (Keyence Inc., 2012).
3. EXPERIMENTAL SETUP AND PROCEDURE

3.1 Numerical analysis of the cantilever

The cantilever beam with piezoelectric actuation—introduced in Section 2.1—was numerically modeled in the ANSYS finite element analysis (FEA) package (Takacs and Rohaľ-Ilkiv, 2012). The beam and the piezoelectric material were meshed using the SOLID5 element with a coupled electromechanical capability to model the piezoelectric effect in the transducers. After creating the solid model, the beam was sectioned to match the placement of the actuators. This sectioned solid model was meshed with a global 8 mm element size and then the nodes were merged in order to connect the actuators with the beam (Ansys Inc., 2009).

Following the modeling and meshing phase, boundary conditions were assigned. All degrees of freedom (DOF) were taken away from nodes located at the fixed end of the beam, to simulate the effect of the clamp (fixed–free boundary condition). The bottom layer of nodes on the piezoceramic transducers (closer to the beam surface) was set to a zero voltage potential, creating one terminal on the actuator. On the inactive transducers, the upper layer of nodes (further from the beam surface) were also set to a zero voltage potential, in order to simulate the short-circuited state. In the active transducers, all nodes in the upper layer were coupled to the node having the smallest index number. The modal analysis was set to utilize the block Lanczos method to extract the first five modes of the beam (Ansys Inc., 2009; Trebuña et al., 2011).

3.2 Exciting the beam to its resonant modes

The smart cantilever beam described in Section 2.1 and its auxiliary components were used to excite the structure to its resonant frequencies. The simplified connection scheme of this system is illustrated in Figure 3. The two piezoceramic actuators were connected to the same signal source, with an opposite electric polarity. The high-voltage signal passed to the piezoelectric actuators is driven by a Midé EL-1225 operational amplifier with a $20 \times V/V$ gain. The amplifier is connected to an analog input terminal, which receives its input signal from a National Instruments PCI-6030E laboratory measurement card. This measurement card is located in a personal computer, running the Matlab xPC Target rapid control prototyping software. The computer is connected via Ethernet to another computer using the Matlab Simulink software suite. The deflection of the tip is measured via a laser triangulation sensor that feeds its signal to the analog input terminal of the measurement card. The signal from the laser sensor was also displayed on an oscilloscope, acting as a reference to fine-tune and find the resonant frequencies of the beam.

To measure the resonance frequencies of the smart beam, a chirp signal was supplied to the actuators in the 0–500 Hz bandwidth. The sampling frequency of the signal was set to 5000, well in the excess of the sampling recommended by Shannon’s theorem. The time domain measurement spanned 300 s. The deformation of the beam was measured by the laser sensor and the signal was recorded and
saved by the xPC Target computer. Subsequently, a FFT transform was performed on this time domain signal. The resulting frequency response of the beam is shown in Figure 4, which depicts the power spectral density estimate of the signal coming from the laser distance measurement device. The figure indicates resonance frequencies in the transversal deformation direction, encompassing the 0–500 Hz bandwidth. The four transversal modes—number 1, 2, 4, 6, 7—are clearly visible, while modes 3 and 5 cannot be detected by this experimental setup. The resonant frequencies obtained from the ANSYS modal simulation are compared to the experimental results in Table 1.

Figure 6 illustrates the Simulink block scheme used to generate sinusoidal signals to the piezoelectric actuators. The main branch of the scheme contains a signal generator, in which the frequency and amplitude of the input signal is set. This signal is then passed to the output port of the measurement card. The input and output data was logged into a file for later processing. The experimentally recorded resonance frequencies were set as the base frequency of the sinusoidal signal supplied to the actuators. In addition to a monitoring oscilloscope, the output from the laser sensor was also read into the block scheme to estimate the deformation amplitudes.

### 3.3 Filming the resonant mode shapes

The camera head was attached to the macro zoom unit with the light projection optics and placed on the free-angle stand. The camera was plugged into the main unit, while an optical fiber cable coupled the light projector with the source inside the main unit. The light source was set to provide a wide spot, in order to cover most of the cantilever beam.

<table>
<thead>
<tr>
<th>Mode:</th>
<th>1.</th>
<th>2.</th>
<th>3.</th>
<th>4.</th>
<th>5.</th>
<th>6.</th>
<th>7.</th>
</tr>
</thead>
<tbody>
<tr>
<td>ANSYS (Hz):</td>
<td>8.1</td>
<td>50.1</td>
<td>103.7</td>
<td>138.9</td>
<td>209.9</td>
<td>270.9</td>
<td>448.4</td>
</tr>
<tr>
<td>Experiment (Hz):</td>
<td>8.1</td>
<td>50.1</td>
<td>100.0</td>
<td>139.1</td>
<td>192.2</td>
<td>274.4</td>
<td>448.1</td>
</tr>
</tbody>
</table>

* a sideways mode
* b twisting mode

All experiments used identical camera settings. The frame rate was set to the native maximum of 4000 FPS, with a 1/8000 s shutter speed. The resolution remained at the maximal 640 x 480 px. The type of event trigger used...
when filming the mode shapes was an end trigger, meaning that the filming was stopped manually and then only the preceding four seconds were saved into the semiconductor memory. A smaller segment of the full four second block was selected to be saved into an uncompressed audio-video interleave (AVI) file with a 15 FPS sampling.

Camera focus and zoom were set manually; none of the viewing angles required the use of a close-up conversion lens. The camera angle and position was re-adjusted using the stand and based on the requirements of the viewing angle. For each of the first 5 resonance modes covered in this paper, four main viewing angles were covered. The beam was filmed from the top, side, front and isometric viewing angles. These angles corresponded to the animated mode shapes extracted from the numerical analysis. Examples of the viewing angles covered by both the numerical analysis and the video imagery are shown in Figure 5 for the fourth resonant frequency and its corresponding mode shape.

4. RESULTS

The video files saved to the hard drive of the main unit were transferred to a personal computer for post-processing in video editing software. The footage was contrasted to the animated mode shapes, obtained from ANSYS simulation results. Each viewing angle was compared and inspected visually for the individual resonant frequencies.

We provide Figures 7 and 8 as an example, illustrating frames from the ANSYS animation in contrast with the high-speed video footage for the second transversal (out of plane) vibration mode of the beam. Note that the static frame grabs presented in this paper cannot fully represent the real vibration levels, the subjective feeling of movement and deformation is better illustrated in the video footage.

The first transversal vibration mode results in the largest deformations. As expected from the ANSYS analysis, the movement is clearly visible from the top, front and isometric viewpoints, but it is not detectable from a side angle\(^2\). The first resonance of the beam is at 8.1 Hz with a period of 0.12 s, this movement is slowed down to 30 s with the native 4000 FPS framerate and a 15 FPS resampling.

Figure 7 shows the second resonance shape of the beam, filmed and animated from the top viewpoint. The upper four images show the beam progressing through its mode shape, while the bottom two images correspond to the video footage of the deformation extremes. As the figures suggest, the deformation levels are relatively small, however still clearly distinguishable. Just as in the animations, the tip of the image is deformed over the reference line, and the middle section under it in Figure 7(e). The situation is reversed in the second extreme position in Figure 7(e). The video file also indicates the position of the vibration node, which is coincident with the simulation results. The same applies for the isometric view shown in Figure 8, therefore the discussion will not be repeated here. As expected from the FEM simulation, deformation is indicated in the sideways view only based on the reflections on the beam surface. The front view footage indicates the deformation of the beam tip and the rest of the beam moving in the opposite direction. Using the previously mentioned recording and resampling framerate, one oscillation cycle is transformed to a 5.3 s long footage.

According to the ANSYS simulation, the third mode shape can be described by a simple in-plane motion. The camera footage reveals that although the shape looks like as expected, the amplitudes are very small and barely detectable. The movement is only visible upon close inspection and a full period is covered in 40 frames or 2.5 s. The fourth vibration mode is a transversal mode again, and it is clearly detectable on the camera footage. The vibration amplitudes are much larger than in the case of the third mode and the resonant nodes are visible as well. The fourth mode is illustrated in Figure 9 from the top, while in Figure 10 from the isometric viewpoint. With the given camera framerate, the period is covered in less than 3 frames, providing about 2 seconds footage at 15 FPS. The next, fifth vibration mode is a twisting mode. The mode is not clear from the top or isometric views; the front view gives only a hint of movement and the mode shape. Almost no movement can be detected on the sixth mode and above, the vibration amplitudes are too small even for this relatively flexible structure.

5. CONCLUSIONS

The visualization of the modal shapes of a smart cantilever beam using a high-speed video camera was presented in
Fig. 7. Second resonance of the smart cantilever beam, viewed from the top viewpoint. The images on the top starting from top left to top right ((a))–(d)) show the animated ANSYS modal analysis results, as the beam is progressing through its second resonance shape. The images on the bottom show screen captures from the high-speed video microscope ((e)–(f)). The white arrows show direction of movement and the thin white line approximates the undeformed beam.

Fig. 8. Second resonance of the smart cantilever beam, viewed from an isometric viewpoint. The images on the top ((a))–(d)) show the results from the ANSYS modal analysis, while the images on the bottom show screen captures from the high-speed video microscope ((e)–(f)) as the beam progresses through its second resonance shape. The white arrows show direction of movement and the thin white line approximates the shape of the undeformed beam.
Fig. 9. Fourth resonance of the smart cantilever beam, viewed from a top viewpoint. The images on the top show ANSYS results, while the images on the bottom show screen captures from the high-speed video microscope. The white arrows show direction of movement and the white circles show the vibration nodes.

Fig. 10. Fourth resonance of the smart cantilever beam, viewed from an isometric viewpoint. The images on the top show ANSYS results, while the images on the bottom show screen captures from the high-speed video microscope. The white arrows show direction of movement and the white horizontal lines show the vibration nodes.
this paper. The video footage retrieved from the camera matched the ANSYS simulation results, essentially creating an effective visualization of the modal behavior of the beam. Because the vibration amplitudes were small compared to the size of the beam, only the first three transversal modes produced useful video footage. The native resolution of the camera allowed filming the resonant vibration only up to the 6th resonant frequency adequately, with full resolution and playback speed. Due to hardware limitations, the 7th mode produced less than 9 frames per second in average. Focusing on the given target object was difficult even using a long-range macro unit, while lightning was also somewhat insufficient.

Advantages of using slow-motion video to characterize modal shapes include the fact that the footage is a direct representation of the deformations not an abstract and processed quantity, its non-invasive nature and easy equipment setup. However, in our opinion, the disadvantages of using a high-speed camera to visualize modal shapes prevail over these advantages. First, even with very flexible structures—like the one presented here—the vibration amplitudes can be small compared to the size of the target object. Although vibration can be detected in the motion picture a little better than isolated frame grabs, the camera only captures the lowest modes reliably. This problem could be partly solved by using a CCD sensor with a much denser pixel resolution. Vibration amplitudes are not the only issue, as the pixel resolution is usually limited by the nominal framerate of the camera. This means, to create at least 15 frames of the vibration period at 4000 FPS, one may only investigate vibration phenomena up to 250 Hz. In addition to this, the exact measurement of deformation to obtain experimental data is somewhat cumbersome and limited by the pixel resolution of the device. Depending on the exact type of high-speed video equipment used, focusing the entire target object and lightning may also be an issue.

According to the work presented in this paper, the use of high-speed video cameras in characterizing vibration mode shapes cannot be recommended in general. Nevertheless, there are certain circumstances in which the method may be suitable:

- A visual confirmation and illustration of mode shapes is required
- The expected vibration amplitudes are large or the target object is very flexible
- Only the lowest mode shapes are in the center of interest
- High-speed video equipment is more readily available than alternatives like ESPI or scanning LDV

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REFERENCES


